Coordination Schemes for the Integration of Transmission and Distribution System Operations

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***Abstract*—Low-voltage distribution networks are emerging as an increasingly important component of power system operations. From a computational standpoint, the proactive utilization of these resources places daunting challenges due to their vast number and the non-linear physics that govern power flow in low-voltage networks. For this reason, a hierarchical approach to the organization of distribution markets which can better cope with computational scalability may be desirable. This paper models various alternatives to the coordination of transmission and distribution system operations, and investigates their relative performance on a small-scale network in terms of allocative efficiency, consistency with physical constraints, and pricing.**

***Index Terms*—distributed resources, distribution system operations, optimal power flow, balancing, second order cone programming**

I. Introduction

The mobilization of distributed resources is emerging as an increasingly important and challenging aspect of power system operations [14]. This paradigm shift towards the proactive management of distributed resources is driven by a number of factors that are influencing the transition of the energy industry, including: (i) the large-scale integration of renewable resources is placing ever-increasing needs on power system flexibility; (ii) residential and commercial demand, which is connected to low-voltage grids, represents the majority of demand-side flexibility [6]; (iii) the proliferation of distributed renewable supply and distributed storage requires an intelligent management of distribution power flows in order to postpone or avoid costly distribution network infrastructure upgrades; (iv) cloud-based communication and control technology appear to offer adequate technological solutions to the computation, communication and control requirements of this transition.

The integration of distributed resources in proactive power system operations poses two major challenges from a modeling and computational perspective: the number of resources is vast, and the physics of distribution networks cannot be adequately represented through linearized power flow models. A direct approach towards the integration of distributed resources involves integrated optimization whereby the transmission system and the distribution system are optimized simultaneously was proposed recently by Caramanis et al. [2], which exploits recent breakthroughs on conic relaxations of optimal power flow [4]. The distribution locational marginal price signals generated from this approach endogenize the value of losses, voltage constraints, complex power flow constraints. In such a scheme, the operations of the distribution system operator (DSO) are effectively absorbed by the transmission system operator (TSO). Hierarchical approaches whereby the DSO reacts to a locational marginal price at the TSO-DSO interface have also been suggested in the literature [15], [3], [11], [16], [8]. Such hierarchical schemes may be required in order to achieve scalability in the mobilization of distributed resources. However, the detailed description and modeling of TSO-DSO coordination has yet to be clarified in the literature. A particularly challenging aspect of TSO-DSO coordination is the extent to which system imbalances can be resolved through the mobilization of distributed reserves while respecting distribution system constraints, and which entity, the TSO or the DSO, should be responsible for this decision.

The SmartNet consortium has proposed various coordination schemes for TSO and DSO operations that aim at the scalable mobilization of distributed resources [5]. The focus of SmartNet is on the *activation* of reserve capacity. How this reserve capacity is committed is a topic that will be addressed separately in future research. The contribution of this paper is to propose models that can be used for the quantitative evaluation of TSO-DSO coordination schemes. The models presented in this paper can be used for assessing the allocative efficiency, the 'proximity' of dispatch to physically compatible solutions and the price signals generated by various TSO-DSO coordination schemes.

The remainder of the paper is organized as follows. Section II presents models for the SmartNet coordination schemes. Section III demonstrates the proposed models on a small-scale test system. Section IV concludes and discusses directions of future research.

II. Modeling TSO-DSO Coordination Schemes

The following models consider the activation of reserves for balancing a real-time deviation in the net load of a transmission or distribution node. It is assumed that reserve capacity has been cleared in earlier markets, and reserve activation bids are offered for upward activation or downward activation in real time. Five schemes will be considered: (i) Centralized common TSO-DSO market, (ii) Decentralized common TSO-DSO market, (iii) Centralized ancillary services

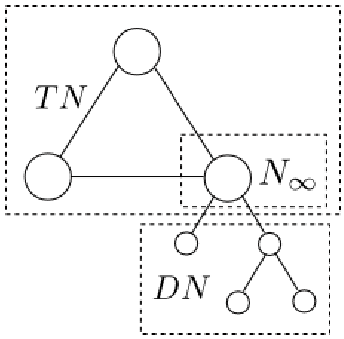

Fig. 1. The set notation used for meshed transmission networks with radial distribution networks.

market, (iv) Local ancillary services market, and (v) shared balancing responsibility. Each of these schemes is described in detail, and for each of the schemes a model is proposed in this section. The topology of the networks considered in this paper consist of meshed transmission networks and radial distribution networks, as indicated in Fig. 1. The notation for the models is summarized in the appendix.

A. Centralized Common TSO-DSO Market

This approach is based on Caramanis et al. [2]. Transmission and distribution network resources are dispatched according to an integrated optimization of the entire system. The goal of the system operator is to minimize the cost of reserve activation.

$$\min \sum_{g \in G} C_g \cdot \Delta p_g + \sum_{i \in DN} C_i^c \cdot \Delta p_i^c + \sum_{i \in DN} C_i^g \cdot \Delta p_i^g \quad (1)$$

The upward marginal cost activation is denoted as C_g for generators connected at the transmission system, and as C_i^c and C_i^g for consumers and generators connected at the distribution system. The upward activation is denoted as Δp_g, Δp_i^c, Δp_i^g for the activation of transmission-level generators, distribution-level consumers, and distribution-level producers respectively. The set of generators is denoted as G, while DN denotes the set of distribution nodes. (see Fig. 1).

The transmission network is represented through linearized power flow equations:

$$f_l = B_l(\theta_n - \theta_m), l = (n,m) \in L, \theta_0 = 0 \quad (2)$$
$$-TC_l \leq f_l \leq TC_l, l \in L, \quad (3)$$
$$\sum_{g \in G_n} (\bar{p}_g + \Delta p_g) + \sum_{l:l=(m,n)} f_l = D_n + \sum_{l \in L:l=(n,m)} f_l + \mathbf{\Delta D_n}(\omega), n \in TN - N_\infty \quad (4)$$
$$\sum_{g \in G_n} (\bar{p}_g + \Delta p_g) + \sum_{l:l=(m,n)} f_l = D_n + \sum_{l \in L:l=(n,m)} f_l + pr_n + \mathbf{\Delta D_n}(\omega), n \in N_\infty \quad (5)$$
$$\Delta p_g \leq R_g, g \in G \quad (6)$$
$$\Delta p_g \geq 0, g \in G \quad (7)$$

The set of transmission lines is denoted as L. The set TN represents the set of transmission nodes, while N_∞ corresponds to the interface of transmission and distribution nodes (see Fig. 1). The susceptance and flow limits of a line are denoted as B_l and TC_l respectively. The decision variables include real power flows f_l, nodal voltage phase angles θ_n, and the transmission-distribution (T-D) interface flow pr_n. There exists a reference bus, indexed by 0, whose bus angle voltage is zero. The set-point of generators from earlier markets is indicated as $\bar{p}_g$, price-inelastic demand is denoted as D_n, and R_g indicates the committed reserve capacity. The random disturbance in net load is indicated as $\mathbf{\Delta D_n}(\omega)$. It is highlighted in bold font because it represents to the disturbance which necessitates the activation of reserves.

The distribution network is represented through a second-order cone programming (SOCP) relaxation, which is tight for radial distribution networks under mild assumptions [4]. The radial network notation is illustrated in [13], Fig. 1. Given a distribution node i, A_i refers to its unique ancestor and C_i to its children.

$$v_i = v_{A_i} + 2(R_i f_i^p + X_i f_i^q) - l_i(R_i^2 + X_i^2), i \in E \quad (8)$$
$$f_i^p - \sum_{j \in C_i} (f_j^p - l_j R_j) - (\bar{p}_i^g + \Delta p_i^g) + (\bar{p}_i^c + \mathbf{\Delta D_i}(\omega) - \Delta p_i^c) + G_i v_i = 0, i \in DN \quad (9)$$
$$-\sum_{j \in C_i} (f_j^p - l_j R_j) - pr_i + G_i v_i = 0, i \in N_\infty \quad (10)$$
$$f_i^q - \sum_{j \in C_i} (f_j^q - l_j X_j) - q_i^g + q_i^c - B_i v_i = 0, i \in DN \quad (11)$$
$$-\sum_{j \in C_i} (f_j^q - l_j X_j) - qr_i - B_i v_i = 0, i \in N_\infty \quad (12)$$
$$(f_i^p)^2 + (f_i^q)^2 \leq v_i l_i, i \in E \quad (13)$$
$$V_i^- \leq v_i \leq V_i^+, i \in DN \cup N_\infty \quad (14)$$
$$\Delta p_i^g \leq R_i^g, i \in DN \quad (15)$$
$$Q_i^{g-} \leq q_i^g \leq Q_i^{g+}, i \in DN \quad (16)$$
$$\Delta p_i^c \leq R_i^c, i \in DN \quad (17)$$
$$Q_i^{c-} \leq q_i^c \leq Q_i^{c+}, i \in DN \quad (18)$$
$$(f_i^p)^2 + (f_i^q)^2 \leq S_i^2, i \in E \quad (19)$$
$$(f_i^p - R_i l_i)^2 + (f_i^q - X_i l_i)^2 \leq S_i^2, i \in E \quad (20)$$
$$l_i \geq 0, \Delta p_i^g \geq 0, \Delta p_i^c \geq 0, q_i^g \geq 0, q_i^c \geq 0, i \in E \quad (21)$$

The voltage magnitude and current magnitude squared are denoted v_i and l_i respectively. The real and reactive power flow over a line are denoted as f_i^p and f_i^q respectively. The set of distribution lines is denoted as E. The resistance and reactance of a distribution line are denoted R_i and X_i respectively. The nodal admittance and susceptance are denoted as G_i and B_i respectively, while $\bar{p}_i^g$ and $\bar{p}_i^c$ indicates reference production and consumption respectively, as determined by forward market clearing. Reactive injections and withdrawals are indicated as q_i^g and q_i^c respectively. Reactive power T-D interface flows are indicated as qr_i. Voltage upper and lower limits are indicated as V_i^+ and V_i^- respectively. Distributed producer and consumer reserve capacities are denoted as R_i^g

and R_i^c respectively. The complex power flow limit of a line is denoted as S_i. The reactive injection and withdrawal limits of producers and consumers are denoted as Q_i^{g+}, Q_i^{g-} and Q_i^{c+}, Q_i^{c-} respectively. The Centralized Common TSO-DSO Market model is the collection of Eqs. (1)-(21).

B. Decentralized Common TSO-DSO Market

This market is modeled by using a *residual* supply function for real power at the balancing market operated by the TSO. This residual supply function is computed by the DSO, which operates its own local market while accounting for its private distribution network constraints [1]. To be more specific, the DSO computes the residual supply function $V_i(pr_i)$ by solving the following problem for different values of pr_i:

$$
\begin{aligned}
V_i(pr_i) = \quad & \min \sum_{i\in DN} C_i^c \cdot \Delta p_i^c + \sum_{i\in DN} C_i^g \cdot \Delta p_i^g \\
& \text{subject to distribution equations} \\
& (8), (9), (11), (12), (13), (14), (15), (16), (17), \\
& (18), (19), (20), (21) \\
(\gamma_i): \quad & -\sum_{j\in C_i} (f_j^p - l_j R_j) + G_i v_i = pr_i, i \in N_\infty
\end{aligned}
$$

Note that this problem needs to be solved *before* the clearing of the TSO balancing market, and is therefore necessarily agnostic about the actual realization of real-power imbalance $\mathbf{\Delta D_i}(\omega)$. For this reason, the real-power imbalance is set equal to zero. In addition, the communication of a detailed demand function $V_i(\cdot)$ may be excessively onerous in terms of computation and communication, therefore it is assumed that the DSO only communicates a *linearization* of the demand function around a set of predetermined points $\bar{pr}_j$:

$$
V_i(pr) \simeq \max_j (V_i(\bar{pr}_j) + \gamma_j \cdot (pr - \bar{pr}_j)),
$$

where $\gamma_j \in \partial V_i(\bar{pr}_j)$ is a subgradient of V_i at the operating point $\bar{pr}_j$. If the market clearing problem of the DSO is convex[1], the function can be seen to be convex. The requisite data that is needed for the linearization of the function can be obtained by the DSO market clearing problem: $V_i(pr_i)$ is simply the objective function of the distribution network subproblem, while γ_i can be obtained from the dual optimal multiplier of the power balance constraint at the interface node.

With the residual supply function (or its linear approximation) in place, the TSO can then solve the following balancing problem:

$$
\begin{aligned}
& \min \sum_{g\in G} C_g \cdot \Delta p_g + \sum_{n\in N_\infty} V_n(pr_n) \\
& \text{subject to transmission equations} \\
& (2), (3), (4), (5), (6), (7)
\end{aligned}
$$

The resulting real power flow on the root node is injected to the DSO system. The distribution network is then dispatched, given the resulting real power injection at the root node. This coordination scheme takes advantage of the fact that the only thing that the TSO and the DSO need to agree on is the real power flow at the interface. This hierarchical control should therefore deliver a near-optimal performance, with reasonable communication requirements between TSO and DSO. Note that the financial roles and responsibilities of each entity are also well defined. The DSO participates in the TSO market through an energy bid, and receives a payment from (or pays to) the TSO for its cleared quantity. The DSO then clears its local market, accounting for its local constraints, and using the previously collected payment from the participation in the TSO auction in order to distribute payments to its local market participants. A similar paradigm has been described in a recent publication by Kristov [10].

[1]This can be achieved if we use the SOCP relaxation of the distribution network subproblem.

C. Centralized Ancillary Services Market Model

In this approach, the TSO clears a market for ancillary services at the transmission level, using resources from the transmission and distribution system, but without accounting for distribution network constraints. In order not to violate distribution network constraints, resources need to be prequalified, in the sense that distribution resources are not offered in the TSO market if they may violate distribution network constraints. This pre-qualification process is not modeled explicitly in this paper, instead it is assumed that prequalification has already been concluded.

The centralized ancillary services market model dispatches the system so as to relieve the imbalance that has occurred by aggregating distributed resources that offer reserve to their root interface node. The TSO solves the following model:

$$
\begin{aligned}
& \min \sum_{g\in G} C_g \cdot \Delta p_g + \sum_{i\in DN} C_i^c \cdot \Delta p_i^c + \sum_{i\in DN} C_i^g \cdot \Delta p_i^g \\
& \text{Subject to transmission equations} \\
& (2), (3), (4), (5), (6), (7) \\
& \text{Subject to distribution equations} \\
& (15), (17), (21) \\
& f_i^p - \sum_{j\in C_i} f_j^p - (\bar{p}_i^g + \Delta p_i^g) + \\
& (\bar{p}_i^c + \mathbf{\Delta D_i}(\omega) - \Delta p_i^c) = 0, i \in DN \\
& -\sum_{j\in C_i} f_j^p - pr_i = 0, i \in N_\infty
\end{aligned} \tag{22}
$$

Note that this coordination scheme ignores real power losses and shunt capacitance losses.

D. Local Ancillary Services Market Model

The local ancillary services market model activates resources depending on where the imbalance occurs. For transmission-level imbalances, the dispatch is obtained as follows:

$$\min \sum_{g \in G} C_g \cdot \Delta p_g + \sum_{i \in DN} C_i^c \cdot \Delta p_i^c + \sum_{i \in DN} C_i^g \cdot \Delta p_i^g$$

Subject to transmission equations

$(2), (3), (4), (5), (6), (7)$

Subject to distribution equations

$(22), (21)$

$$f_i^p - \sum_{j \in C_i} f_j^p - (\bar{p}_i^g + \Delta p_i^g) + (\bar{p}_i^c - \Delta p_i^c) = 0, i \in DN$$

$$\Delta p_i^g \leq RLAST_i^g, i \in DN$$

$$\Delta p_i^c \leq RLAST_i^c, i \in DN$$

where $RLAST_i^{g/c}$ corresponds to the ancillary services capacity that is available for transmission system balancing from node i of the distribution network.

For a distribution system imbalance, the dispatch is obtained by fixing the amount of real power injection from the transmission system to $\bar{pr}_i$:

$$\min \sum_{i \in DN} C_i^c \cdot \Delta p_i^c + \sum_{i \in DN} C_i^g \cdot \Delta p_i^g$$

Subject to distribution equations

$(8), (9), (11), (12), (13), (14), (16), (18), (19), (20), (21)$

$$-\sum_{j \in C_i} (f_j^p - l_j R_j) - \bar{pr}_i + G_i v_i = 0, i \in N_\infty$$

$$\Delta p_i^g \leq RLASD_i^g, i \in DN$$

$$\Delta p_i^c \leq RLASD_i^c, i \in DN$$

where $RLASD_i^{g/c}$ corresponds to the ancillary services capacity that is available for distribution system balancing from the distribution network.

E. Shared balancing responsibility

The shared balancing responsibility requires that the TSO clear transmission-level imbalances by using transmission-level resources *only*, and the DSO clear distribution-level imbalances by using distribution-level resources *only*.

The TSO subproblem is modeled as

$$\min \sum_{g \in G} C_g \cdot \Delta p_g$$

Subject to transmission equations

$(2), (3), (4), (6), (7)$

$$\sum_{g \in G_n} (\bar{p}_g + \Delta p_g) + \sum_{l:l=(m,n)} f_l = D_n + \sum_{l \in L:l=(n,m)} f_l + \bar{pr}_n + \mathbf{\Delta D_n}(\omega), n \in N_\infty$$

Note that the objective function only involves transmission-level resources, and the root injection to the distribution network is fixed to the result of the forward market set-point $\bar{pr}_n$, meaning that the TSO does not coordinate with the DSO.

The DSO subproblem is modeled as

$$\min \sum_{i \in DN} C_i^c \cdot \Delta p_i^c + \sum_{i \in DN} C_i^g \cdot \Delta p_i^g$$

Subject to distribution equations

$(8), (9), (11), (12), (13), (14), (15), (16), (17), (18), (19), (20), (21)$

$$-\sum_{j \in C_i} (f_j^p - l_j R_j) - \bar{pr}_i + G_i v_i = 0, i \in N_\infty$$

The objective function only involves distribution-level resources, and the root injection of real power to the distribution network is fixed to the set-point of the forward market $\bar{pr}_i$, implying no coordination between the DSO and the TSO.

Since the interface power flow is fixed to the result of the forward market, the operations of the TSO are fully decoupled from those of the DSO. The resulting balancing actions are feasible, provided the local imbalances do not exceed the local reserves. However, the resulting balancing action may be more costly than necessary, because the reserve resources cannot be pooled.

III. Numerical illustration

This section tests the models presented in the previous sections on a small-scale system, in order to illustrate their differences. The studied network is presented in Fig. 2. The network consists of three nodes, each of which further consists of five distribution nodes.

The full data of the model is available in the following link. In summary, the system consists of two thermal units at the transmission level. The unit located in node 1 has a marginal cost of 10 €/MWh, and a capacity of 390 MW. The unit located in node 2 has a marginal cost of 20 €/MWh and a capacity of 150 MW. There is an inelastic demand of 350 MW in location 1. Each distribution tree has identical line characteristics and identical resources are connected to each distribution tree. Each distribution node is connected to a distributed aggregated producer of 85 MW and a distributed aggregated consumer of 80 MW. Aggregated flexible consumers with bid quantities of 50 MW and valuations ranging from 0 €/MWh up to 19.1 €/MWh are connected to each distribution node. Thus, each of the three distribution trees can offer up to 250 MW of upward reserve (if flexible demand is fully consuming), serves a price-inelastic demand of 400 MW, and zero-cost aggregated distributed production of 425 MW (which could also offer reserve) is connected to each distribution tree.

The following discussion will concentrate on upward reserve activation, in order to keep the analysis targeted. Consider the following commitment of reserve capacity in the system, which is obtained a result of a forward reserve capacity auction[2]: (i) Generator 2 offers 149.1 MW, at an activation cost of 20 €/MWh; (ii) Consumer 15 offers 49.1 MW, at an activation cost of 19.1 €/MWh; (iii) consumer 25 offers 6.3

[2]In the present example, the reserve commitment is based on a perfectly coordinated reserve capacity auction, as described by Caramanis et al. [2].

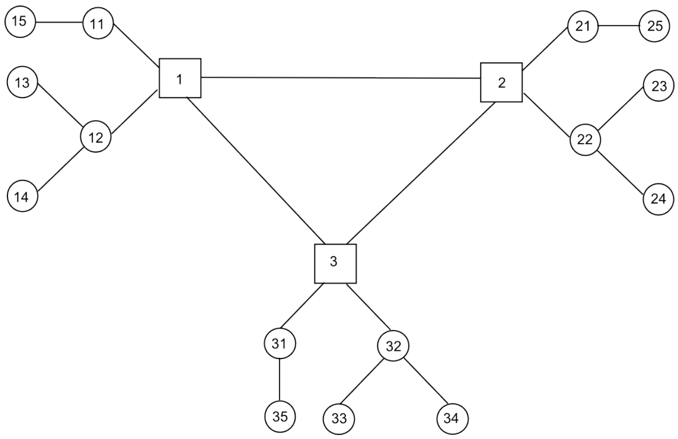

Fig. 2. The network analyzed in the numerical illustration of section III.

Interface	Point 1	Point 2	Point 3
1	7.4	18.8	19.2
2	15.0	15.0	15.1
3	15.0	17.7	19.8

TABLE I
RESIDUAL SUPPLY FUNCTION MARGINAL COST (IN €/MWH) FOR THE DECENTRALIZED COMMON TSO-DSO MARKET MODEL.

MW, at an activation cost of 15 €/MWh; and (iv) consumer 34 offers 49.1 MW, at an activation cost of 19 €/MWh.

Consider an imbalance of -100 MW in node 3, which is caused by an increase of demand in node 3 that equals $\mathbf{\Delta D_3}(\omega) = +100$ MW.

A. Centralized Common TSO-DSO Market

This coordination scheme resolves the imbalance by simultaneously accounting for transmission and distribution constraints. The imbalance is resolved by dispatching demand response to the following levels: (i) consumer 15 provides 49.1 MW of activated reserve[3], and its consumption level is 0.9 MW; (ii) consumer 25 offers 6.3 MW of activated reserve, its consumption level is 0.7 MW; (iii) consumer 34 offers 40.2 MW of activated reserve, its consumption level is 9.8 MW. The real-time price becomes 19.3 €/MWh uniformly across the entire transmission network. The price in the distribution network is in the range of 17.9-19.3 €/MWh.

B. Decentralized Common TSO-DSO Market

The resolution of the DSO unperturbed model with three points of approximation of the residual supply function yields the function shown in table I. Note that the marginal cost function of the interface is increasing, as expected when the DSO problem is convex, and as required for market clearing of the TSO market.

The resulting dispatch of the TSO resources is *identical* to that obtained by the centralized common TSO-DSO model, with generator 1 producing 390 MW, generator 2 producing nothing, and the reserves required for balancing sourced from the interfaces. The balancing price of the transmission market is 19.7 €/MWh. It should be noted that the sourcing of the reserve shifts slightly, relative to the sourcing obtained in the centralized common market model. In the decentralized model, the interface nodes balance the system with the following injections (the negative sign indicates that the flow is from the distribution pocket to the transmission system): $pr_1 =$ -20.49 MW, $pr_2 =$ -20.84 MW, and $pr_3 =$ -18.67 MW. In the centralized model, the injections are $pr_1 =$ -22.80 MW, $pr_2 =$ -23.00 MW, and $pr_3 =$ -14.20 MW.

One therefore observes that the decentralized approach shifts sourcing of reserve from pockets 1 and 2 to pocket 3. This can be attributed to the fact that the true marginal cost of activating distributed resources is only approximate, not exact, in the decentralized model. Notice, however, that the total amount of real power reserve which is activated should be equal in both cases, because in both cases the contribution of transmission-level resources to clearing the imbalance is identical. By consequence, there is a slight loss in consumer benefit and balancing prices at the transmission system are also slightly different from those of the centralized common market model. Note also that, by construction, this market-clearing method will not violate system constraints.

C. Centralized Ancillary Services Market Model

This coordination scheme dispatches resources by ignoring the distribution network constraints. In particular, the imbalance is resolved by dispatching distributed demand response resources to the following levels: (i) the consumption level of consumer 15 is 13.5 MW; (ii) the consumption level of consumer 25 is 0.7 MW; and (iii) the consumption level of consumer 34 is 0.9 MW.

The real-time price becomes 19.1 €/MWh, uniformly across the entire transmission network, which is slightly lower than that of the centralized common TSO-DSO market model, and is due to the underestimation of losses. Note that there is no DLMP in this coordination scheme, instead distributed resources receive the transmission-level price.

The dispatch is not feasible, because losses in the distribution network are ignored. Solving a feasibility restoration problem (analogous to a phase-I procedure in linear programming), one finds that a *positive* activation of 4.0 MW is necessary in location 15 in order to restore feasibility.

Note that a significant amount of the balancing has shifted from consumer 34 (in the centralized common TSO-DSO market model) to consumer 15 (in the centralized ancillary services model).

D. Local Ancillary Services Market Model

In this model the DSO clears a local market for reserve before a transmission-level market is cleared. The local market commits half of the reserve capacity for use by the local DSO, with the other half (the more expensive half, since the DSO reserve market clears first) being made available to the TSO.

[3]Activated reserve is measured as the difference between the production level in the co-optimization reservation problem and the production after the imbalance appears and is resolved with the co-optimization model.

Thus, reserve capacity is allocated as follows. (i) The TSO can access reserves from generator 2 up to 149.1 MW at an activation price of 20 €/MWh, reserves from consumer 15 up to 24.55 MW at an activation price of 19.1 €/MWh, reserves from consumer 25 up to 3.15 MW at an activation price of 15 €/MWh, and reserves from consumer 34 up to 24.55 MW at an activation price of 19 €/MWh. (ii) The DSO of feeder 1 can access reserves from consumer 15 up to 24.55 MW at an activation price of 19.1 €/MWh. (iii) The DSO of feeder 2 can access reserves from consumer 25 up to 3.15 MW at an activation price of 15 €/MWh. (iv) The DSO of feeder 3 can access reserves from consumer 34 up to 24.55 MW at an activation price of 19 €/MWh.

The real-time price at the transmission level is 20.0 €/MWh. The reason is that the distribution network reserves are activated up to their full capacity (only half of the total distributed reserves are available to the TSO), and the generator with marginal cost 20 €/MWh needs to be activated. This sets the price of 20 €/MWh for balancing in the transmission network.

The imbalance is resolved by dispatching demand response resources and transmission-level generation to the following levels: (i) generator 2 produces 39.7 MW, (ii) consumer 15 withdraws 25.4 MW, (iii) consumer 25 withdraws 3.8 MW, and consumer 34 withdraws 25.4 MW. The resulting dispatch violates physical constraints, and the resolution of a phase-I feasibility restoration requires an excess production of 3.9 MW at location 34.

One notable feature of this model is that the same resource can be activated in opposite directions, depending on the imbalance for which it is activated. For example, if there is a positive transmission-level imbalance and a negative distribution-level imbalance, a distributed resource may be activated upwards by the TSO and downwards by the DSO.

E. Shared Balancing Responsibility Model

This model separates the dispatch in the transmission and the distribution networks by fixing the linking variable of the two networks, which is the real power flow at the interface. The value of the real power at the interface in this numerical example is fixed to the value obtained from a forward reserve capacity auction.

The real-time price at the transmission level is 20.0 €/MWh. The reason is that the TSO has no access to distribution network reserves, and therefore the generator with marginal cost 20.0 €/MWh needs to be activated. This sets the price of 20.0 €/MWh for balancing in the transmission network. The imbalance is resolved by dispatching generator 2 at 100.0 MW.

The resulting dispatch results in a feasible power flow. This can be understood by the fact that the shared balancing responsibility model will not violate feasibility at the transmission network, because it is consistently accounting for real power (since there are no overlooked distribution network losses), and it will not violate feasibility at the distribution network since it does not simplify the distribution network constraints. However, in general this feasibility will come at a relatively high activation cost since resources of the transmission and distribution networks are not pooled. This is evident in table II, where generator costs increase dramatically (outweighing additional consumer benefits), and exceed the activation costs of any other coordination scheme.

The relative performance of the studied coordination schemes in terms of allocative efficiency, physical feasibility, and price, are summarized in table II.

	Gen. cost [$]	Consumer benefit [$]	Constraint violation [MW]	Transm. price [€/MWh]
CCM	3900.0	212.4	0.0	19.3
CAS	3900.0	283.7	4.0	19.1
LAS	4693.3	1026.0	3.9	20.0
SBR	5900.0	200.9	0.0	20.0
DCM	3900.0	202.2	0.0	19.7

TABLE II
SUMMARY STATISTICS OF THE DIFFERENT COORDINATION SCHEMES. THE COORDINATION SCHEME INITIALS STAND FOR CENTRALIZED COMMON MARKET (CCM), DECENTRALIZED COMMON MARKET (DCM), CENTRALIZED ANCILLARY SERVICES MARKET (CAS), LOCAL ANCILLARY SERVICES MARKET (LAS), AND SHARED BALANCING RESPONSIBILITY (SBR).

IV. CONCLUSIONS

This paper proposed models for quantifying five proposals of TSO-DSO coordination which have recently been proposed by the SmartNet project consortium. The following conclusions can be drawn from the specific case study presented in this paper: (i) The Centralized Common Market model sets the first-best standard in terms of allocative efficiency, however it is challenging to implement due to the large scale of the optimization problem and the communication requirements. (ii) The Decentralized Common Market model strikes a balance between efficiency and computational / communication tractability. This is achieved by exploiting the fact that the TSO and DSO need only agree on the amount of real power flowing over the T-D interface. (iii) The Local Ancillary Services model is dominated by the Centralized Ancillary Services model in terms of allocative efficiency. (iv) The Shared Balancing Responsibility model will not violate physical constraints, however it appears to be the least efficient solution.

Future research will focus on decomposition methods for tackling the Common Market model. In particular, it is appealing to consider a two-way communication of TSOs and DSOs with cloud-based infrastructure, where distributed computations can be performed and communicated back to devices [14]. Alternatively, peer-to-peer algorithms may be suitable for overcoming communication requirements [9]. In case a hierarchical coordination scheme is adopted, Generalized Nash Equilibrium [12], [7] appears to offer an appropriate theoretical framework for analyzing the resulting interactions of TSOs and DSOs, since TSO decisions affect the feasible set of DSO control actions and vice versa.

Appendix

This appendix summarizes the notation used in the paper.

Sets

G / G_n: the set of generators / generators located in bus n
TN: set of transmission nodes
DN: set of distribution nodes
DN_i: set of distribution nodes located under interface i
L: set of transmission lines
N_∞: the set of buses at the transmission / distribution interface
E: set of distribution network edges

Parameters

$C_g(\cdot)$: cost of generator g as a convex function of power production (assumed linear in the text)
B_l: susceptance of transmission line l
TC_l: flow limit of line l
P_g^-/P_g^+: min / max capacity limit of generator g
D_n: real power demand in node n
R_i / X_i: resistance / reactance of distribution line i
G_i / B_i: shunt conductance / shunt susceptance of distribution node i
$C_i^g(\cdot)$: cost of generator in distribution node i as a convex function of power production (assumed linear in the text)
$C_i^c(\cdot)$: consumer valuation in distribution node i as concave function of power consumption (assumed linear in the text)
V_i^- / V_i^+: minimum / maximum voltage limit of distribution or interface node i
P_i^{g-} / P_i^{g+}: minimum / maximum real power limit of generator at distribution node i
P_i^{c-} / P_i^{c+}: minimum / maximum real power limit of consumer at distribution node i
Q_i^{g-} / Q_i^{g+}: minimum / maximum reactive power limit of generator at distribution node i
Q_i^{c-} / Q_i^{c+}: minimum / maximum reactive power limit of consumer at distribution node i
$\bar{p}_g$: real power production cleared in earlier markets for generator g
$\bar{p}_i^g$: real power production cleared in earlier markets at distribution node i
$\bar{p}_i^c$: real power consumption cleared in earlier markets at distribution node i
$\Delta D_n(\omega)$: real power imbalance in node n
$R_g/R_i^g/R_i^c$: committed reserve of generator g / generator in distribution node i / consumer in distribution node i
$RLAST_i^{g/c}$: the amount of reserves that is available in location i of the distribution network from a generator or consumer resource, and is used for resolving transmission-level imbalance

Variables

Δp_g: reserve activation of generator g
f_l: real power flow over transmission line l
θ_n: bus angle of transmission bus n
pr_n / qr_n: root real power / reactive power injection in T&D interface node n
v_i: voltage magnitude squared at distribution node i
f_i^p / f_i^q: real / reactive power flow over distribution line i
l_i: current magnitude squared of distribution line i
Δp_i^g: real power reserve activation at distribution node i
Δp_i^c: real power reserve activation at distribution node i
q_i^g: reactive power production at distribution node i
q_i^c: reactive power consumption at distribution node i

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